HERIOT WATT UNIVERSITY

Heriot-Watt University

Radiation Heat Transfer Thermodynamics 2

Dr.Peter Cumber

Radiation Heat Transfer
Thermodynamics 2
By Peter Cumber

Prerequisites

- Interest in thermodynamics
- Some ability in calculus (multiple integrals)
- Good understanding of conduction and convection

1. Introduction

1.1 Conduction and convection heat transfer

2. When is radiation heat transfer important?

3 Surface Radiation

3.1 Reflected Radiation

3.2 Black Surfaces

3.3 Radiation leaving a surface

4. Fundamental properties of radiation

4.1 The Spectrum

1. Introduction

Heat transfer between two bodies is the transfer of energy due to a temperature difference. There are three modes of heat transfer, conduction, convection and radiation heat transfer. To appreciate the differences between the different modes of heat transfer a superficial overview of conduction and convection heat transfer is given below.

1.1 Conduction and convection heat transfer

Conduction and convection were considered in depth in previous heat transfer modules. Conduction heat transfer occurs in solid materials, liquids and gas phases although it is often only significant compared to other modes of heat transfer in the solid phase.

Consider steady heat transfer through a wall, see the figure below,

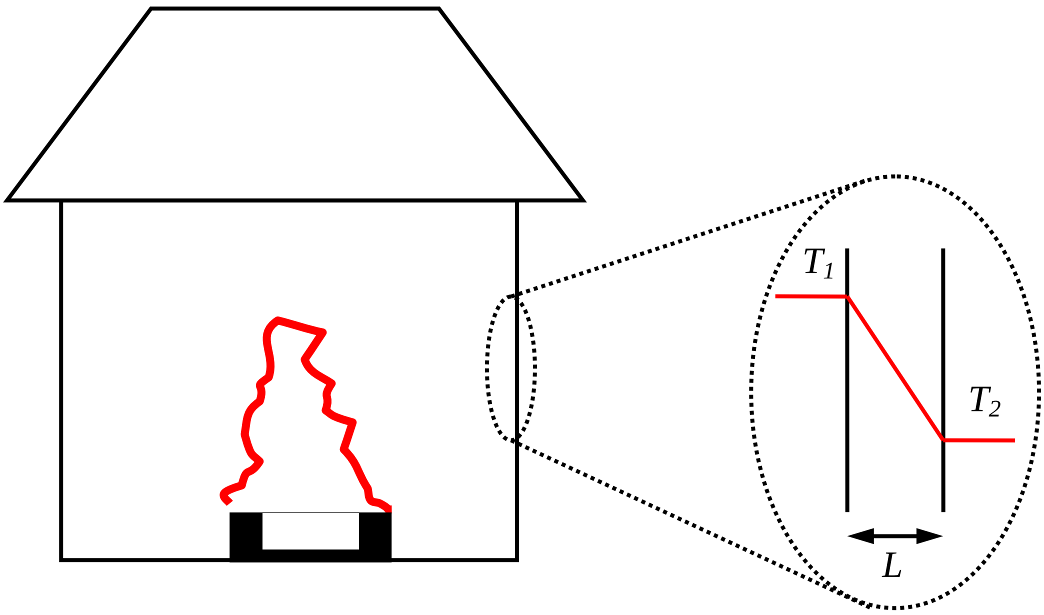

Figure 1.1 – Conduction heat transfer through a wall.

The heat transfer per unit area, the heat flux is given by,

$$\dot{q}_{cond} = -\frac{k}{L}\left(T_2 - T_1\right)$$

where k denotes the thermal conductivity. Convection heat transfer occurs in liquids and gases and is due to the bulk movement of a hot or warm fluid. Convection heat transfer is often characterised as natural or forced convection. Natural convection is fluid motion induced by a density change due to heat transfer. Forced convection occurs when the fluid motion is induced by a fan or pump.

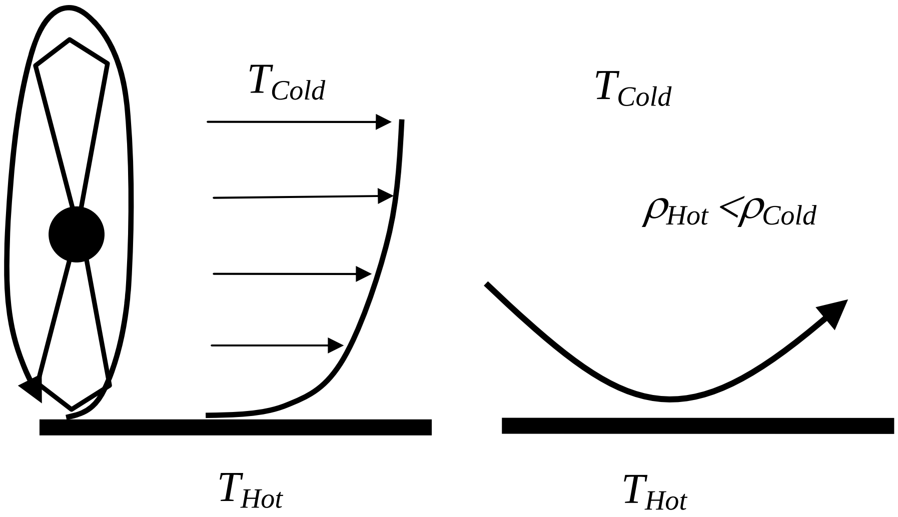

Figure 1.2 – Forced and natural convection from a hot plate below a cold.

In both cases the convection heat transfer is modelled by Newton's law of cooling,

$$\dot{q}_{conv} = h\left(T_{Hot} - T_{Cold}\right)$$

where *h* is the convection heat transfer coefficient and is dependent on the geometry, fluid properties and in the case of forced convection, the bulk fluid speed.

2. When is radiation heat transfer important?

All bodies with a temperature above absolute zero emit energy in the form of electromagnetic waves, this is radiation heat transfer. Electromagnetic waves can pass through solid materials, liquids and the gas phase. In addition electromagnetic waves can pass through a vacuum. Radiation does not require a medium i.e. molecules organised in a lattice /(solid) or molecules more "loosely connected" such as in liquids and gases. The best example of radiation heat transfer in a vacuum is the transfer of energy as electromagnetic waves from the sun to the earth.

Let us consider when radiation heat transfer could be significant compared to other heat transfer mechanisms. In the liquid and gas phase heat transfer by conduction can in most cases be considered negligible. For the present if we consider a hot surface at a temperature, T_{surf} and the surrounding air temperature is T_{amb}, the net radiation heat transfer from the surface is given by the equation,

$$\dot{q}_{rad} = \varepsilon\sigma\left(T_{surf}^4 - T_{amb}^4\right)$$

where ε is the total emissivity of the surface and σ is the Stefan Boltzmann constant. These terms will be explained and the T^4 temperature dependence of the radiation heat transfer justified in later sections.

Consider the ratio,

$$\frac{\dot{q}_{rad}}{\dot{q}_{conv}} = \frac{\varepsilon\sigma\left(T_{surf}^4 - T_{amb}^4\right)}{h\left(T_{surf} - T_{amb}\right)}$$

A reasonable order of magnitude estimate for the heat transfer coefficient *h* for natural convection from a hot surface below a cool fluid is,

$$h = 20\frac{W}{m^2K}$$

The Stefan Boltzmann constant in S.I. units is,

$$\sigma = 5.67\times10^{-8}\frac{W}{m^2K^4}$$

and the total emissivity takes a value between,

$$0 < \varepsilon < 1$$

If we take the ambient air temperature to be,

$$T_{amb} = 293K$$

we can plot the ratio,

$$\frac{\dot{q}_{rad}}{\dot{q}_{conv}}$$

as a function of the surface temperature.

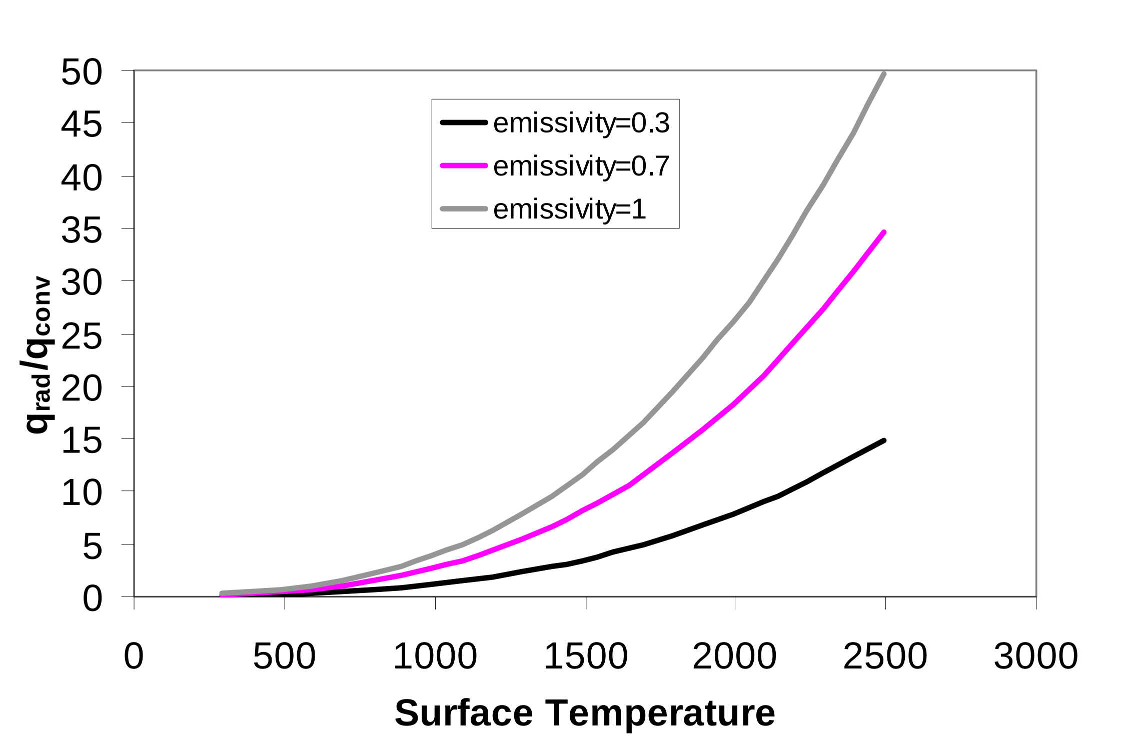

Figure 2.1 - Ratio of radiation to convection heat transfer from a hot surface.

From the analysis above it is seen that radiation heat transfer compared to convective heat transfer is significant for surface temperatures above 600K.

Radiation heat transfer is important when analysing:

- High temperature processes '(i.e. combustion)
- No other modes of heat transfer are possible (transfer through a vacuum)
- When accurate calculations of heat transfer is very important. For example predicting the life of sensitive electronic components

3 Surface Radiation

The radiation incident on a surface is denoted as *H* and is sometimes called the **irradiance**.

Consider radiation incident on a surface next to a source of radiation such as a fire, radiation traverses the space between the fire and the surface in the form of electromagnetic waves and impinge on the surface. Three possible interactions with the surface are possible.

- A proportion of incident radiation is reflected from the surface,
- A proportion is transmitted through the solid material
- A proportion is absorbed by the material

See the figure below.

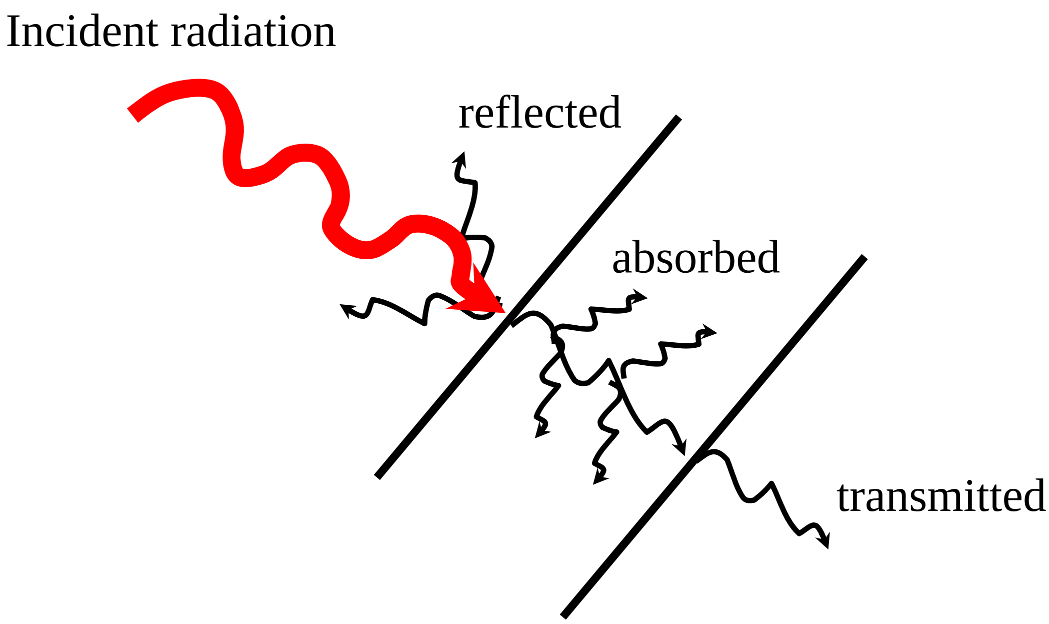

Figure 3.1 Radiation Incident to a surface.

It is possible to define three properties of relevance to analysing the radiation heat transfer to and from the surface.

$$\alpha = \frac{incident\ radiation\ absorbed}{total\ incident\ radiation}$$

$$\rho = \frac{incident\ radiation\ reflected}{total\ incident\ radiation}$$

$$\tau = \frac{incident\ radiation\ transmitted}{total\ incident\ radiation}$$

Energy conservation gives,

$$\alpha + \rho + \tau = 1$$

Most solid materials can be characterised as opaque and the transmissivity is zero.

$$\alpha + \rho = 1$$

3.1 Reflected Radiation

Reflected radiation can be characterised as being either specular or diffuse in nature. Specular reflection occurs for smooth surfaces such as mirrors, or polished metals. The physical laws of optics can be applied to specular surfaces such as the angle of reflection equals the angle of incidence. Diffuse reflection occurs when the surface is uneven and rough. In this case multiple reflections occur in all directions, with no preferential direction of reflection.

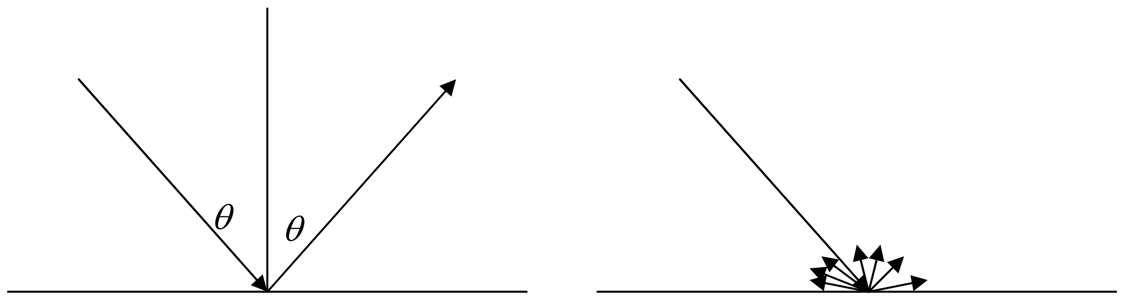

Figure 3.2 – Specular reflection and diffuse reflection from a surface.

Diffuse and specular surfaces represent two limits with real surfaces exhibiting some combination of the two limiting cases.

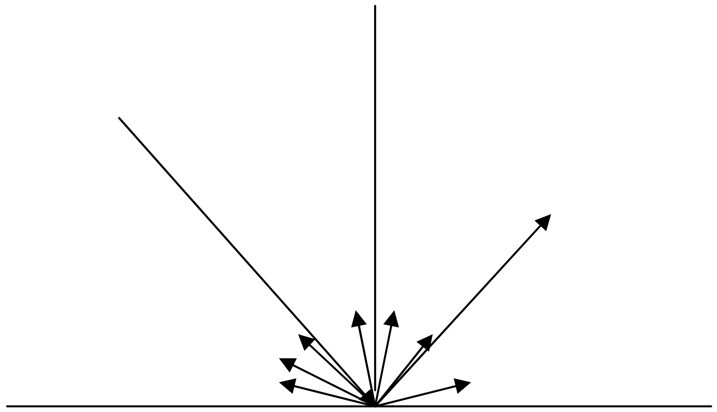

Figure 3.3 – Reflection from a real surface.

3.2 Black Surfaces

Where the entire radiation incident on a surface is absorbed i.e. there is no reflected or transmitted radiation,

$$\alpha = 1$$

it is called a black surface or black body. This is similar to a black hole in space as no energy incident on the surface "escapes", it is all absorbed. It is possible to achieve very high absorbtivities using matt black coatings, but a perfectly black surface is not possible in practice.

When discussing nearly black surfaces some care must be taken as the absorptivity is considered across the full spectrum important in thermal radiation analysis. The thermal radiation spectral range is several orders of magnitude larger than the visible range, i.e. what you or I can see. This can lead to seemingly surprising results, for example the absorptivity of white paper is 0.97. Similarly snow has a high absorptivity in the thermal radiation spectrum and a high reflectivity in the visible spectrum.

3.3 Radiation leaving a surface

The total radiation leaving a surface is called the radiosity, denoted *B*. The radiation leaving a surface is separated into the reflected radiation and the emitted radiation.

$$B = \rho H + \varepsilon E_b \qquad \textbf{(3.1)}$$

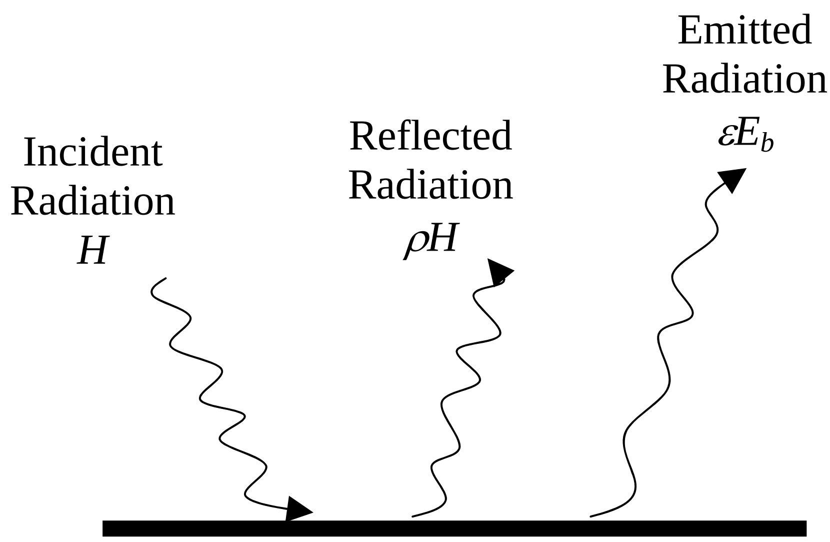

Figure 3.4 - Radiation leaving a surface.

The reflected radiation has already been discussed above and by definition of the reflectivity is,

$$\rho H$$

the emitted radiation is the energy leaving a surface due to it having a temperature above absolute zero. As can be seen from equation (**3.1**) the emitted radiation is calculated from a product, εE_b.

ε denotes the emissivity of the surface as defined below,

$$\varepsilon = \frac{Emitted\ radiation}{Theoretical\ Max\ emitted\ radiation}$$

By its definition it can be seen that,

$$0 \le \varepsilon \le 1$$

E_b denotes the theoretical maximum emitted radiation and is called the black body emissive power. This is a strong function of temperature and will be considered in more detail in the following sections.

If we consider a black surface,

$$\rho = \tau = 0, \ \ \alpha = 1$$

an argument based on thermodynamic equilibrium between the surface and the surrounding air if they are at the same temperature shows that the emissivity of a black body is,

$$\varepsilon = \alpha = 1$$

i.e. black bodies emit radiation at the maximum possible energy level for the surface temperature. Similarly it is possible to show for a **grey surface** the term used for a surface with an absorptivity of less than one, that

$$\varepsilon = \alpha < 1$$

This is called **Kirchhoff's radiation law**. The emissivity is often temperature dependent, but a table of emissivities at ambient temperature is given below. Some of the emissivities quoted are quite counter-intuitive.

Material	Emissivity
Brass - Polished	0.03
Copper -Polished	0.03
Copper -Oxidised	0.8
Mild Steel –Polished	0.07
Mild Steel –Galvanized	0.3
Mild Steel –Rusted	0.8
Brick –Fire Clay	0.7
Glass – Polished	0.94
Paper – White	0.97
Paint – White gloss	0.9
Paint – Black gloss	0.9
Paint – Black matt	0.97
Paint – Aluminium	0.3-0.6
Water	0.95

Table 3.1 – Emissivity of a number of materials at ambient temperature.

Having established when radiation heat transfer might be important and defined a number of surface properties important in the analysis of radiation heat transfer let us have a look at some of the fundamental properties in radiation heat transfer.

4. Fundamental properties of radiation

In convection and conduction heat transfer the fundamental quantity of interest is the heat flux, denoted *q*, which has units of power per unit area, with S.I. units of W/m^2. For radiation heat transfer a more fundamental quantity than the heat flux needs to be defined as radiation heat transfer is directional, (it can be thought of as energy travelling in straight lines or rays). In addition the electromagnetic waves that form the basis of radiation heat transfer oscillate in a plane perpendicular to the direction of travel, see the figure below.

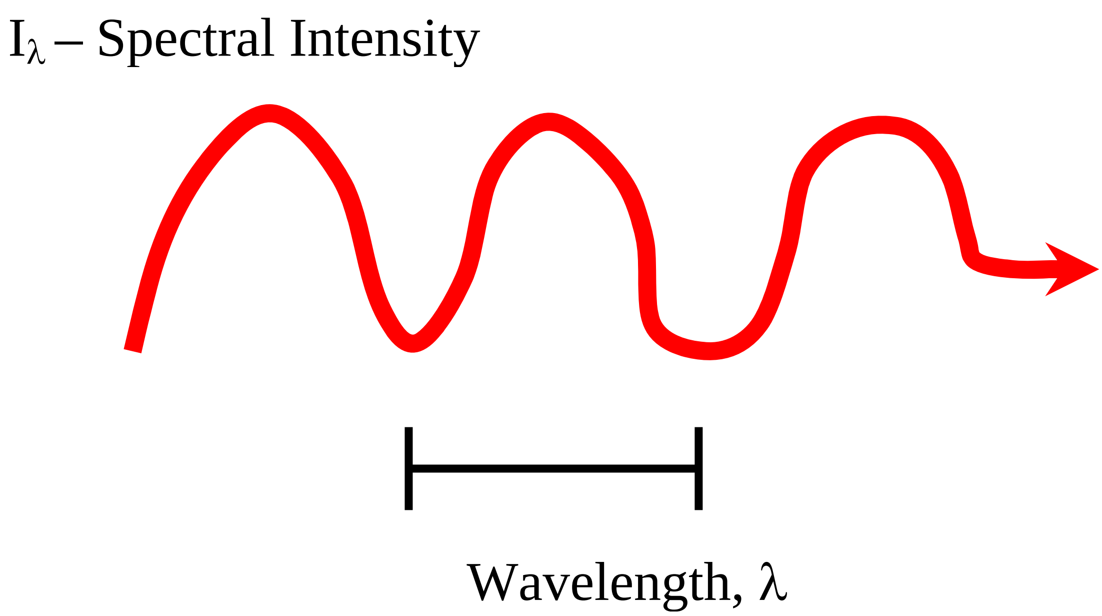

Figure 4.1 – The spectral intensity with a wavelength λ.

This leads to the definition of the spectral intensity,

$$I_\lambda(\underline{x}, \underline{n})$$

where λ is the wavelength, $\underline{x}$ is the location in space and $\underline{n}$ is a set of direction cosines that give the orientation of the ray or "pencil of radiation". The spectral variable is sometimes specified in terms of the wave number,

$$\eta = \frac{1}{\lambda}$$

or specified as the frequency,

$$\nu = \frac{c_0}{\lambda}$$

where c_0 is the speed of light in a vacuum. To avoid confusion when considering spectral properties we will always use the wavelength as the independent spectral variable. Wavelength is given in units of length and is typically expressed in terms of 10^{-6} metres, micrometres or more often referred to as "microns".

4.1 The Spectrum

Although the wavelength for electromagnetic waves varies from very short wavelengths, 10^{-8} metres up to 10^{10} metres, see the figure below, our interest is in the spectral window where the majority of the energy is located. This is the thermal radiation region of the spectrum.

$$0.4\mu m \le \lambda_{thermal\ radiation} \le 1{,}000\ \mu m$$

For reference the visible portion of the spectrum for the human eye is

$$0.4\mu m \le \lambda_{visible} \le 0.7\ \mu m$$

4.1.1 Planck Function

The maximum possible spectral intensity is dependent on the temperature and is given by the function originally proposed by Max Planck.

$$I_{\lambda,b}(T) = \frac{2C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$$

Note C_1 and C_2 are physical constants. The *b* subscript is used to signify a 'black body' spectral intensity, the maximum possible intensity at a given wavelength.

The figure below shows the Planck function for a range of temperatures.

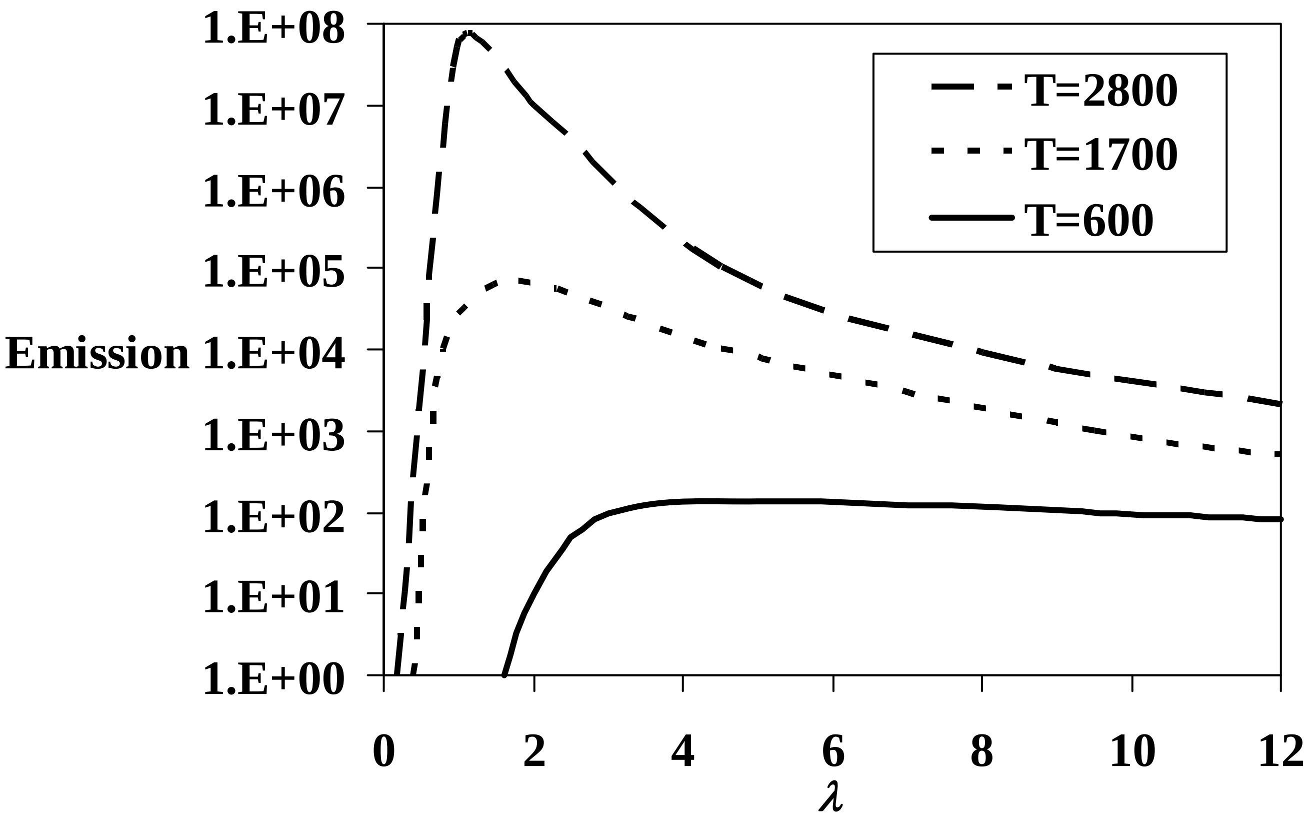

Figure 4.2 – Spectral intensity for a range of temperatures.

Two points are of particular interest.

A. The nonlinear dependence on temperature i.e. small increases in temperature give relatively large increases in intensity, note the logarithmic scale for emission..

B. The maximum intensity shifts to the left as the temperature increases.

As the temperature increases the maximum intensity moves towards the visible portion of the spectrum and the surface begins to glow red changing to a yellow and eventually glowing white as the temperature increases further.

The location of the maximum intensity as a function of wavelength is approximately given by **Wien's displacement law**,

$$\lambda_{\max} T = C_3$$

where

$$C_3 = 2898\ \mu m\ K$$

This means the maximum spectral intensity enters the visible spectral range for a temperature of,

$$T = \frac{C_3}{0.7} = 4{,}140\ K$$

however experimentally surfaces begin to glow red at temperatures as low as 500K.

4.1.2 The integrated intensity

Generally for engineering applications the spectral intensity is not of interest. The integrated intensity,

$$I\left(\underline{x}, \underline{n}\right) = \int_0^\infty I_\lambda\left(\underline{x}, \underline{n}\right) d\lambda$$

is of more interest as it can be used to derive the radiation heat flux.

Similar to the spectral black body intensity, the integrated black body intensity can be defined as,

$$I_b\left(\underline{x}, \underline{n}\right) = \int_0^\infty I_{\lambda, b}\left(\underline{x}, \underline{n}\right) d\lambda$$

$$= \int_0^\infty \frac{2C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)} d\lambda$$

This can be simplified considerably if we use the substitution,

$$\xi = \frac{C_2}{\lambda\, T} \Rightarrow \lambda = \frac{C_2}{\xi\, T}$$

$$d\xi = -\frac{C_2}{\lambda^2\, T} d\lambda \Rightarrow d\lambda = -\frac{\lambda^2 T}{C_2} d\xi = -\frac{C_2}{T\xi^2} d\xi$$

considering the limits of integration for λ and ξ.

$$\lambda \to 0 \quad \xi \to \infty$$

and

$$\lambda \to \infty \quad \xi \to 0$$

Making the substitution into the integral,

$$I_b\left(\underline{x},\underline{n}\right) = \int_0^\infty \frac{2C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)} d\lambda = \int_\infty^0 \frac{2C_1 T^5 \xi^5}{C_2^5\left(e^\xi - 1\right)}\left(-\frac{C_2}{T\xi^2}\right) d\xi$$

This can be simplified to,

$$I_b\left(\underline{x},\underline{n}\right) = \frac{2C_1}{C_2^4} T^4 \int_0^\infty \frac{\xi^3}{e^\xi - 1} d\xi$$

The integral is an improper integral that can be evaluated (honest) to give,

$$\int_0^\infty \frac{\xi^3}{e^\xi - 1} d\xi = \frac{\pi^4}{15}$$

This gives the result

$$I_b\left(\underline{x},\underline{n}\right) = \frac{2C_1 \pi^4}{15 C_2^4} T^4$$

or more concisely,

$$I_b\left(\underline{x},\underline{n}\right) = \frac{\sigma}{\pi} T^4$$

where

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

This is an important result as it shows that the radiation is strongly dependent on temperature, small increases in temperature can give a big increase in radiation levels.

4.1.3 Solid angle

The total or integrated intensity has units of power per unit area per unit solid angle. Solid angle is a term unfamiliar to most students unfamiliar with radiation heat transfer. If we consider a hot surface and radiation is being emitted from a point on the surface, then the radiation is emitted in all directions. This can be considered as radiation is emitted from a hemisphere surrounding the point. The hemisphere can be considered to have a unit radius. The unit hemisphere has a surface area of 2π or equivalently a solid angle of 2π steridians. Steridian is the S.I. unit of solid angle. If we wanted to consider the intensity field over a smaller solid angle as shown in the figure below

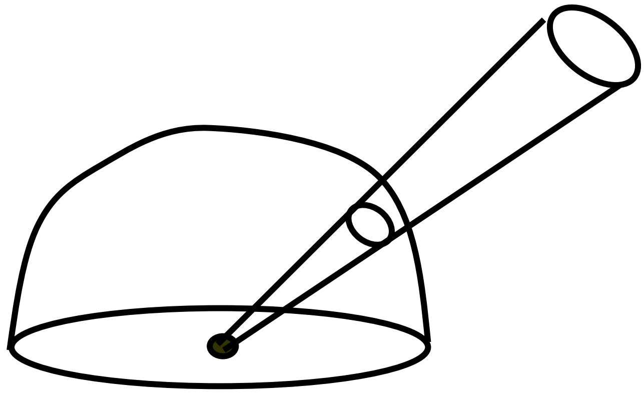

Figure 4.3 – Pencil of radiation defining a solid angle on the unit hemisphere.

The solid angle is the area on the unit hemisphere defined by projecting the pencil of radiation back onto the unit hemisphere. This allows us to make the connection between the total intensity and the radiation heat flux.

4.1.4 Lambert's Cosine Law

A geometric analysis shows that the emitted radiation heat transfer is dependent on the angle of incidence, θ in the figure below. If we consider a total intensity from a small surface dA the energy flow in the direction θ is related to the total intensity and the projected area of dA, denoted dA_p, where dA_p is perpendicular to the orientation of the ray.

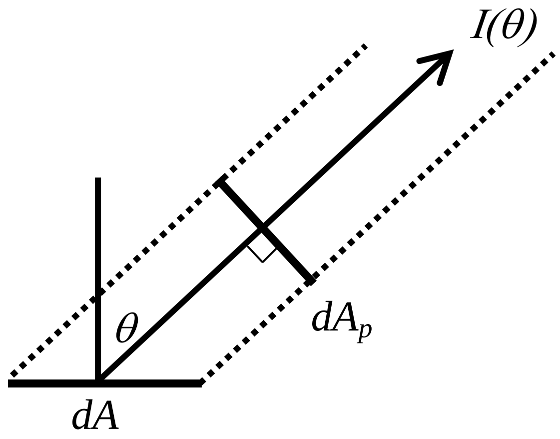

Figure 4.4 – Projected area as a function of angle of incidence.

This leads onto the definition of the emitted radiation heat flux, consider the triangle formed by the area and projected area in the figure below.

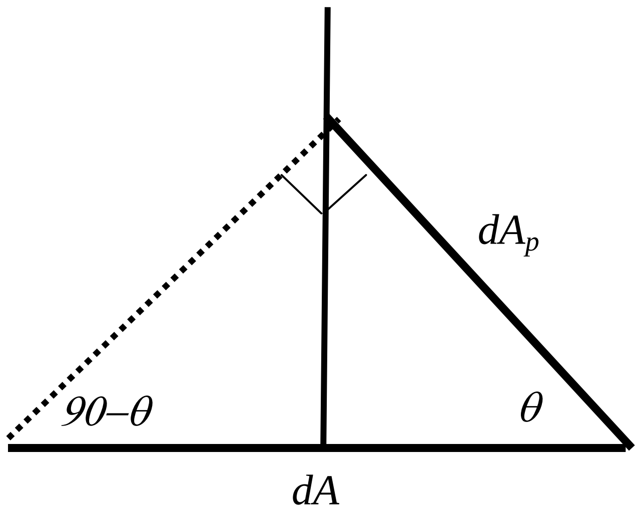

Figure 4.5 – Geometry defined by the projected area and the angle of incidence.

From the figure above trigonometry gives us that

$$dA_p = \cos\theta \; dA$$

This is called Lambert's cosine law and enables us to derive an expression for the radiation heat flux.

$$\varepsilon E_b(\underline{x}) = \int_{\Delta\Omega} I(\underline{x}) \cos\theta \; d\Omega$$

$$E_b = \int_{\Delta\Omega} I_b \cos\theta \, d\Omega = \frac{\sigma T^4}{\pi} \int_{\Delta\Omega} \cos\theta \, d\Omega$$

where the integration is over the unit hemisphere denoted as $\Delta\Omega$ and $d\Omega$ denotes a solid angle on the unit hemisphere. Expressing the orientation of the ray using spherical coordinates, θ is the angle of incidence and φ is the angle of rotation, see the figure below.

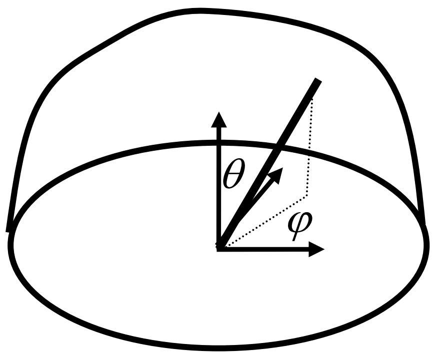

Figure 4.6 – Definition of spherical coordinates on the unit hemisphere.

In spherical coordinates the elemental solid angle is specified as,

$$d\,\Omega = \sin\theta \, d\theta \, d\varphi$$

This leads to the emitted flux integral as,

$$\varepsilon E_b = \int_0^{2\pi} \int_0^{\pi/2} I(\underline{x}) \cos\theta \sin\theta \, d\theta \, d\varphi$$

where cos θ is included because of Lambert's cosine law. If we consider emission from a black body then we have the double integral

$$E_b = \int_0^{2\pi} \int_0^{\pi/2} \frac{\sigma T^4}{\pi} \cos\theta \sin\theta \, d\theta \, d\varphi$$

$$= \frac{\sigma T^4}{\pi} \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\varphi$$

$$= \frac{\sigma T^4}{\pi} \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{2} \sin 2\,\theta \; d\theta \; d\varphi$$

$$\boxed{E_b = \sigma T^4}$$

We now have sufficient background in radiation heat transfer to do some engineering analysis of some thermal systems